

2. A continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0, & x < -2 \\ \frac{x+2}{6}, & -2 \leq x \leq 4 \\ 1, & x > 4 \end{cases}$$

(a) Find $P(X < 0)$. (2)

(b) Find the probability density function $f(x)$ of X . (3)

(c) Write down the name of the distribution of X . (1)

(d) Find the mean and the variance of X . (3)

(e) Write down the value of $P(X = 1)$. (1)



3. A robot is programmed to build cars on a production line. The robot breaks down at random at a rate of once every 20 hours.

(a) Find the probability that it will work continuously for 5 hours without a breakdown. **(3)**

Find the probability that, in an 8 hour period,

(b) the robot will break down at least once, **(3)**

(c) there are exactly 2 breakdowns. **(2)**

In a particular 8 hour period, the robot broke down twice.

(d) Write down the probability that the robot will break down in the following 8 hour period. Give a reason for your answer. **(2)**



4. The continuous random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} k(x^2 - 2x + 2) & 0 < x \leq 3 \\ 3k & 3 < x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{9}$ (4)

(b) Find the cumulative distribution function $F(x)$. (6)

(c) Find the mean of X . (3)

(d) Show that the median of X lies between $x=2.6$ and $x=2.7$ (4)



6. (a) Define the critical region of a test statistic. (2)

A discrete random variable X has a Binomial distribution $B(30, p)$. A single observation is used to test $H_0 : p = 0.3$ against $H_1 : p \neq 0.3$

(b) Using a 1% level of significance find the critical region of this test. You should state the probability of rejection in each tail which should be as close as possible to 0.005 (5)

(c) Write down the actual significance level of the test. (1)

The value of the observation was found to be 15.

(d) Comment on this finding in light of your critical region. (2)



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7. A bag contains a large number of coins. It contains only 1p and 2p coins in the ratio 1:3

(a) Find the mean μ and the variance σ^2 of the values of this population of coins. (3)

A random sample of size 3 is taken from the bag.

(b) List all the possible samples. (2)

(c) Find the sampling distribution of the mean value of the samples. (6)

Lined area for student answers, consisting of 20 horizontal lines.



